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STUDIES IN STATISTICAL SIGNAL PROCESSING

Annual Report

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This is an annual report on Contract AF-88-0327, for the period July 1, 1988 to June 30, 1989. Section I provides a brief overview of our work, while the remaining sections describe in some detail our recent results on efficient factorization of structured matrices. polynomial zero-location, and recursive layer peeling. The appendices list publications and other activities during the last year. The abstract of the recently completed Ph.D. dissertation of J. Chun is also appended to this report.

1. INTRODUCTION

The primary objective of our research is to develop efficient and numerically stable algorithms for nonstationary signal processing problems by understanding and exploiting special structures, both deterministic and stochastic, in the problems. We also strive to establish and broaden links with related disciplines, such as cascade filter synthesis, scattering theory, numerical linear algebra, and mathematical operator theory for the purpose of cross fertilization of ideas and techniques. These explorations have led to new results both in estimation theory and in these other fields, e.g., to new algorithms for triangular and QR factorization of structured matrices, new techniques for root location and stability testing, and new recursions for orthogonal polynomials on the unit circle and the real line as well as on other curves.

For several years, the guiding principle in these studies has been the concept of generalized displacement structure (Lev-Ari and Kailath (1986)), which generalized and subsumed our earlier work on Toeplitz-oriented displacement structure (Kailath, Kung and Morf, (1979); see also Lev-Ari and Kailath (1984)). A related notion of

displacement structure has also emerged from recent work of Heinig and Rost (1984,1987) of East Germany. While they are aware of our work, and make some attempts to relate to it, their approach and methodology are significantly different from ours. In particular, they focus only on the problem of inversion of structured matrices via algebraic methods, while our work has primarily addressed triangular factorization of such matrices, and our approach is based on a generating function characterization of matrices. The triangular factorization problem is in many senses more fundamental than inversion, and has more consequences for signal processing, linear algebra, operator theory and other fields. In fact, recently we were able to reduce the inversion problem for structured matrices to the factorization of certain block-matrices with structured blocks (see Chun and Kailath (1989)). This result also confirms and clarifies an earlier observation [see Lev-Ari and Kailath (1984)] on the relation between efficient inversion and efficient factorization of structured matrices: only some of the structured matrices that admit an efficient factorization procedure can also be efficiently inverted.

The generating function approach, which was introduced in the Ph.D. research of Lev-Ari (1983), also suggests a natural *system-theoretic* interpretation of the theory, which allows a study of various problems in ... m theory, such as minimal realization, Padé approximation, control design, and a variety of root distribution (stability) problems for polynomials. This approach also unveils the great flexibility in the computational details of the factorization procedure for structured matrices. [In contrast, the approach taken by Heinig and Rost leads to a single procedure for (the inversion of) every particular type of structured matrices]. In particular, we were able to recognize some classical root location procedures, such as the Schur-Cohn and the

Routh-Hurwitz algorithms, as particular instances of our factorization procedure. Moreover, by exploiting the flexibility in our prototype procedure we obtained new alternatives to these classical algorithms, with reduced computational requirements (see Lev-Ari, Bistritz and Kailath (1989)).

Perhaps the most prominent system-theoretic aspect of our efficient factorization techniques is that they can be interpreted as recursive identification procedures for certain *lossless cascade models*. For instance, the classical Schur algorithm is also a procedure for step by step identification and 'peeling' of the layers of a transmission-line with a piecewise constant characteristic impedance [Bruckstein and Kailath (1987)]. Such layered models have been used for quite some time in oil exploration and in marine seismography. They involve two scalar signals propagating in opposite directions; consequently, the characteristics of the model can be captured by a single scalar input-scattering function. Schur's original formulation of his algorithm was, in fact, in terms of this scattering function.

We have recently begun to extend our methods to *multichannel cascade models*, which involve involve multiple signals propagating both in the forward and in the opposite (backward) direction. Since such models are represented by *matrix scattering functions*, it would seem that the corresponding layer-peeling procedures need to be rederived in matrix (or block) form. This is certainly possible (see, e.g., Delsarte, Genin and Kamp (1979)), but results in the introduction of computation-intensive matrix operations, such as the evaluation of matrix square roots. In contrast to this approach, we have succeeded in obtaining layer-peeling procedures that involve only elementary (2×2) circular and hyperbolic rotations, and therefore require only scalar

computations. Furthermore, our procedures can be implemented in pipelined parallel processing hardware (such as systolic arrays). Consequently, the throughput of such implementations is independent of the number of channels (i.e., the number of forward and backward signals flowing through the model).

More details on this problem, and some of the results mentioned above, are given in the following sections.

2. FACTORIZATION OF STRUCTURED MATRICES

Our early work on factorization and inversion of Toeplitz and close-to-Toeplitz matrices led us to the observation that for certain matrices the *displacement matrix*

$$\nabla_Z R := R - ZRZ^*, \quad Z = [\ \delta_{i,j+1}\ ]_{i,j=0}^{n}$$

has low rank. Notice that Z has unity elements on the first subdiagonal and zeros elsewhere. Consequently, the displacement matrix $\nabla_Z R$ is the difference between the matrix R and the matrix ZRZ^* obtained by displacing R one step along the main diagonal. In particular, the displacement rank (i.e., the rank of $\nabla_Z R$) is 2 for both Toeplitz matrices and inverses of Toeplitz matrices. We have shown in previous work (largely supported by AFOSR) that the displacement concept is a key tool for developing fast algorithms of many kinds, including factorization and inversion of Toeplitz and near-Toeplitz matrices, as well as fast (generalized Levinson and Schur) algorithms for solving linear systems with such coefficient matrices. Not surprisingly, these results led naturally to cascade orthogonal structures for the prediction of nonstationary processes (Lev-Ari and Kailath (1984)). We have also found that the same concept is tightly connected to the more general problem of cascade filter

synthesis in network theory and digital filtering as well as to a variety of inverse scattering problems (some references are Rao and Kailath, (1984, 1985), Bruckstein and Kailath (1987)).

Later we extended the concept of displacement structure to a very broad family of structured matrices, including Hankel matrices and their inverses, sums of Toeplitz and Hankel matrices and several others (Lev-Ari and Kailath, (1986)). The generalized displacement of a matrix $\mathbf{R}$, is defined as $d(\mathbf{Z},\mathbf{Z})\mathbf{R}$ where

$$d(\mathbf{A},\mathbf{B})\mathbf{R} := \sum_{k,l=0}^{N} d_{k,l}\, \mathbf{A}^k\, \mathbf{R}\, (\mathbf{B}^*)^l \ , \tag{1}$$

and the asterisk $(*)$ denotes Hermitian transpose (complex conjugate for scalars).

The concept of displacement structure and its properties are more conveniently described in terms of *generating functions*. The generating function of a matrix $\mathbf{R}$ is a power series in two complex variables, viz.,

$$R(z,w) := [1 \quad z \quad z^2 \ ...]\mathbf{R}[1 \quad w \quad w^2 \ ...]^* \tag{2}$$

The displacement $d(\mathbf{Z},\mathbf{Z})\mathbf{R}$ of a matrix has the generating function $d(z,w)R(z,w)$, where

$$d(z,w) = \sum_{k,l=0}^{N} d_{k,l}\, z^k (w^*)^l \tag{3a}$$

Thus the generating function of a Hermitian matrix with a displacement structure has the form

$$R(z,w) = \frac{G(z)\, J\, G^*(w)}{d(z,w)} \tag{3b}$$

where J is any constant nonsingular Hermitian matrix. The triple $\{d(z,w),G(z),J\}$

is called a generator of $R(z,w)$, since it uniquely determines the generating function $R(z,w)$.

We have extended our previous work (Lev-Ari and Kailath (1984)) on efficient factorization of matrices with displacement structure to accommodate the generalized displacement $d(\mathbf{Z},\mathbf{Z})\mathbf{R}$, and we have shown (Lev-Ari and Kailath (1986), Lev-Ari (1989)), that efficient factorization of $\mathbf{R}$ is possible if, and only if, there exist power series $\phi(z),\psi(z)$ (with arbitrary radii of convergence) such that

$$d(z,w) = \phi(z)\phi^*(w) - \psi(z)\psi^*(w) \ . \tag{4}$$

To obtain the factorization of $\mathbf{R}$ one has to propagate the recursion (with $G_0(z) \equiv G(z)$)

$$(z-\zeta_i)G_{i+1}(z) = G_i(z)\Theta_i(z) \qquad i = 0,1,2, \dots \tag{5a}$$

where

$$\Theta_i(z) = \left\{ I - \frac{d(z,\tau_i)}{d(z,\zeta_i)d(\zeta_i,\tau_i)} \ JM_i \right\} U_i \tag{5b}$$

$$M_i \ := \ G_i^*(\zeta_i)R_i^{-1}(\zeta_i,\zeta_i)G_i(\zeta_i) = M_i^* \ , \tag{5c}$$

U_i is any constant matrix such that

$$U_i \ J \ U_i^* = J \ , \tag{5d}$$

and τ_i is any complex constant such that

$$d(\tau_i,\tau_i) = 0 \ . \tag{5e}$$

The standard choice of the *extraction points* $\{\zeta_i\}$, i.e., $\zeta_i = 0$, produces triangular factorizations; other choices can be useful in root-location and filter synthesis

procedures (see, e.g., Deprettere and Dewilde (1980), Vaidyanathan and Mitra (1984)).

This algorithm requires $O(n^2)$ computations to factor a structured $n \times n$ matrix $\mathbf{R}$ in the form $\mathbf{R} = \mathbf{LDL}^*$, in contrast to the conventional LDL^* algorithm which requires $O(n^3)$ operations to factor an arbitrary $n \times n$ matrix. The i-th element of the diagonal matrix $\mathbf{D}$ and the i-th column of the lower triangular matrix $\mathbf{L}$ are determined by the coefficients of the power series expansion of $G_i(z)$.

The displacement representation (3) also leads to the following fundamental result: for every $d(z,w)$ of the form (4) and for every (finite) matrix $\mathbf{R}$,

$$In \left\{ d(\mathbf{Z},\mathbf{Z})\mathbf{R} \right\} = In \left\{ d(\mathbf{Z},\mathbf{Z})\mathbf{R}^{-\natural} \right\} \qquad (6)$$

where the reversed matrix $\mathbf{R}^{-\natural}$ is obtained by transposing $\mathbf{R}^{-1}$ with respect to the anti-diagonal, namely the *reversed matrix* $\mathbf{Q}^{\natural}$ is defined as

$$\mathbf{Q}^{\natural} := \tilde{\mathbf{I}} \, \mathbf{Q}^T \, \tilde{\mathbf{I}} \qquad (7a)$$

where $\mathbf{Q}$ can be any square matrix, the superscript T denotes the conventional (non-Hermitian) transpose, and $\tilde{\mathbf{I}}$ is the anti-diagonal unity matrix, viz.,

$$\tilde{\mathbf{I}} := \begin{bmatrix} & & & 1 \\ & 0 & \cdot & \\ & \cdot & & \\ & \cdot & 0 & \\ 1 & & & \end{bmatrix} \qquad (7b)$$

The fundamental result (6) implies that $\mathbf{R}$ and $\mathbf{R}^{-\natural}$ have the same displacement structure. It does not tell us, however, how to relate the parametrizations of generators of $\mathbf{R}$ and of $\mathbf{R}^{-\natural}$. Such relations are known for Toeplitz matrices and give rise to the Levinson algorithm and the Gohberg-Semencul formula. They have been extended by Lev-Ari and Kailath (1984) to close-to-Toeplitz matrices, i.e., to matrices with displacement structure involving $d(z,w) = 1 - zw^*$.

We have recently shown that our factorization procedure can be extended to matrices for which the generalized displacement $\mathbf{R} - \mathbf{FRF}^*$ has low rank, where $\mathbf{F}$ can be an arbitrary lower triangular matrix (Chun and Kailath (1989)). [When $\mathbf{F}$ is a lower-triangular *Toeplitz matrix* the new procedure reduces to the one reported in (Lev-Ari and Kailath (1986))]. Moreover, we have shown how to embed the problem of inverting a structured matrix in a factorization problem of a *double size*. More specifically, given $\{\mathbf{G}, J\}$ such that

$$\mathbf{R} - \mathbf{FRF}^* = \mathbf{G}J\mathbf{G}^* \tag{8a}$$

we introduce the block matrix

$$\mathbf{M} := \begin{bmatrix} \mathbf{R} & \mathbf{I} \\ \mathbf{I} & \mathbf{O} \end{bmatrix} . \tag{8b}$$

Next we look for matrices $\widetilde{\mathbf{F}}, \widetilde{\mathbf{G}}$ that satisfy the generalized displacement equation

$$\begin{bmatrix} \mathbf{R} & \mathbf{I} \\ \mathbf{I} & \mathbf{O} \end{bmatrix} - \begin{bmatrix} \mathbf{F} & \mathbf{O} \\ \mathbf{O} & \widetilde{\mathbf{F}} \end{bmatrix} \begin{bmatrix} \mathbf{R} & \mathbf{I} \\ \mathbf{I} & \mathbf{O} \end{bmatrix} \begin{bmatrix} \mathbf{F} & \mathbf{O} \\ \mathbf{O} & \widetilde{\mathbf{F}} \end{bmatrix}^* = \begin{bmatrix} \mathbf{G} \\ \widetilde{\mathbf{G}} \end{bmatrix} J \begin{bmatrix} \mathbf{G} \\ \widetilde{\mathbf{G}} \end{bmatrix}^* \tag{8c}$$

which is an embedding of (8a). The factorization procedure can now be used to obtain a displacement representation of the Schur complement of the (2,2) block in $\mathbf{M}$, viz.,

$$\begin{bmatrix} \mathbf{O} & \mathbf{O} \\ \mathbf{O} & -\mathbf{R}^{-1} \end{bmatrix} - \begin{bmatrix} \mathbf{F} & \mathbf{O} \\ \mathbf{O} & \widetilde{\mathbf{F}} \end{bmatrix} \begin{bmatrix} \mathbf{O} & \mathbf{O} \\ \mathbf{O} & -\mathbf{R}^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{F} & \mathbf{O} \\ \mathbf{O} & \widetilde{\mathbf{F}} \end{bmatrix}^* = \begin{bmatrix} \mathbf{O} \\ \mathbf{H} \end{bmatrix} J \begin{bmatrix} \mathbf{O} \\ \mathbf{H} \end{bmatrix}^* .$$

In other words, we obtain a generator of $\mathbf{R}^{-1}$. A comprehensive account of this approach for the case $\mathbf{F} = \mathbf{Z}$ is available in [Chun and Kailath (1989)], where we have also shown how to modify this method to obtain extensions of the Gohberg-Semencul formula. The Ph.D. dissertation of Chun (1989) describes several other

applications of suitable embedding procedures.

3. BEZOUTIANS AND EFFICIENT ZERO-LOCATION

One application of the notion of generalized displacement structure is the construction of *Bezoutian matrices*. Originally, such matrices were associated with stability (and zero-location) tests with respect to the unit circle and the imaginary axis. We have extended this notion to structured matrices whose generating functions have the form

$$B(z,w) = \frac{p(z)p^*(w) - p^\#(z)[p^\#(w)]^*}{d(z,w)} \tag{14a}$$

where

$$d(z,w) := \alpha + \beta z + (\beta w)^* + \delta z w^* , \tag{14b}$$

and the sharp (#) denotes a suitably defined polynomial transformation. The remarkable property of such matrices is that most of their elements vanish *except the elements in the leading $n \times n$ principal submatrix*, where $n := \deg p(z)$. This submatrix, which we denote by $\mathbf{B}$, has full rank (i.e., *rank* $\mathbf{B} = n$) if, and only if, $p(z)$ and $p^\#(z)$ are coprime. Furthermore, the inertia of the Bezoutian matrix $\mathbf{B}$ (i.e., the number of its positive, zero and negative eigenvalues) serves to locate the zeros of the polynomial $p(z)$ with respect to the following partition of the complex plane,

$$\Omega_+ = \{z \; ; \; d(z,z) > 0\}$$
$$\Omega_0 := \{z \; ; \; d(z,z) = 0\} \tag{15}$$
$$\Omega_- := \{z \; ; \; d(z,z) < 0\}$$

More specifically the inertia of $\mathbf{B}$ indicates how many zeros are shared by $p(z)$ and

$p^{\#}(z)$ and how many of the remaining zeros are in Ω_+ and in Ω_-. Our fast factorization procedure makes it possible to determine the inertia of a Bezoutian matrix in $O(n^2)$ operations, starting with $p(z)$ and $p^{\#}(z)$, and without explicit evaluation of the elements of **B**. For Bezoutians on the imaginary axis and the unit circle our formulation leads (among other possibilities) to the Routh-Hurwitz and Schur-Cohn tests, and serves to delimit the family of $O(n^2)$ polynomial zero-location procedures (Lev-Ari, Bistritz and Kailath (1989)).

4. RECURSIVE LAYER PEELING

The fundamental factorization procedure (5) for structured matrices, viz.,

$$(z - \zeta_i)G_{i+1}(z) = G_i(z)\Theta_i(z)$$

is, at the same time, also a *layer-peeling procedure*. Starting with $G_0(z)$, which we can interpret as boundary data for a layered medium, we identify an elementary layer with chain-scattering matrix $\Theta_0(z)$, then "peel" it off to obtain $G_1(z)$, the boundary data for the rest of the medium (with the first layer removed), and repeat the same procedure again and again. Such layer-peeling recursions have been used in cascade filter synthesis (see, e.g., Dewilde, Vieira and Kailath, (1978); Vaidyanathan and Mitra (1984)), in inverse scattering (Bruckstein and Kailath (1987)), zero-location (Lev-Ari, Bistritz and Kailath (1987)), and model-order reduction (see, e.g., Genin and Kung (1981)).

The classical work of Schur (1917) forms the basis for much of the subsequent work on layer peeling procedures. Schur's algorithm associates a sequence of so-called *reflection coefficients*, all with magnitude bounded by unity, with every passive

scattering function, i.e., a function $f(z)$ that is analytic and bounded by 1 in the unit disc. In particular, if $f(z)$ is an *all-pass function*, which means that $|f(e^{j\theta})| = 1$ for all θ, then Schur's algorithm produces a *finite* sequence of reflection coefficients $\{k_i \; ; \; 0 \le i \le n\}$, where $|k_n| = 1$ and $|k_i| < 1$ for $0 \le i \le n - 1$.

Another property of the algorithm is that starting with a passive scattering function it generates a sequence of such functions. This is the essence of the layer peeling method: a single step of the Schur algorithm applied to a passive medium leaves a medium with the same property, which makes it possible to apply the same *step again and again*. A *single step* of the Schur algorithm corresponds to the removal (or peeling) of an elementary lossless two-port. Thus, the algorithm produces a discrete transmission-line model, whose input scattering function is $f(z)$ (Fig. 1):
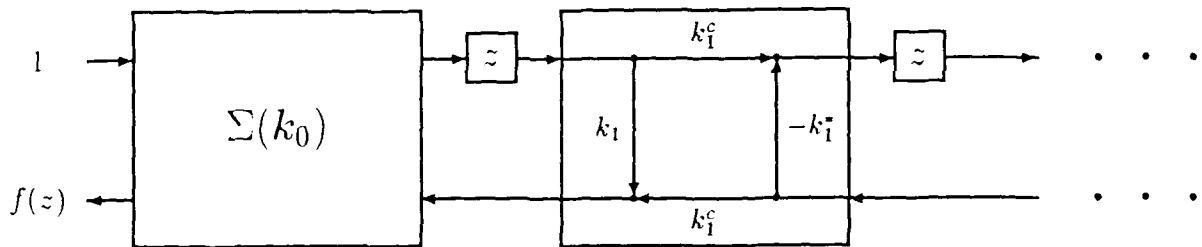

Figure 1. Transmission-line model associated with the Schur algorithm.

In addition to the recursive characterization of passivity via the constraint on the magnitude of the reflection coefficients, Schur also introduced an operator-norm characterization of passivity: he proved that for any function $f(z)$ that is analytic in

the unit disc we can construct an infinite lower-triangular Toeplitz matrix whose first column consists of the coefficients of the power series expansion of $f(z)$, viz.,

$$\mathbf{L}(f) = \begin{bmatrix} f_0 & & & \\ f_1 & f_0 & & \mathbf{0} \\ f_2 & f_1 & \cdot & \\ \cdot & & \cdot & \cdot \\ \cdot & & & \cdot & \cdot \end{bmatrix}$$ (16a)

such that

$$\sup_{|z| < 1} |f(z)| = \|\mathbf{L}(f)\|_2 \leq 1$$ (16b)

where $\|A\|_2$ denotes the conventional (spectral) norm of a matrix A, i.e.,

$$\|A\|_2 := \sup_x \frac{\|Ax\|_2}{\|x\|_2}$$ (16c)

and $\|x\|_2$ denotes the Euclidean (l_2) norm of a vector x.

Schur's algorithm was later extended by Cohn (1922) to functions with *poles in the unit disc*, but only to rational "all-pass" functions, i.e., to functions $f(z)$ of the form

$$f(z) = \lambda \frac{p^{\#}(z)}{p(z)}$$ (17a)

where $p^{\#}(z)$ denotes the conjugate reversal operation, viz.,

$$p^{\#}(z) = z^{deg\ p(z)} \left[p(1/z^*) \right]^*$$ (17b)

The now well-known Schur-Cohn test associates with each such function[†] a finite sequence of reflection coefficients, some of which have magnitudes larger than 1. Moreover, it has been shown (e.g., using the properties of Bezoutians on the unit disc) that the number of poles of $f(z) = p^{\#}(z)/p(z)$ inside the unit-disc equals the number

---

[†] Assuming $p(z)$ has no zeros at $z = 0$, and applying the algorithm to $f(z)/\lambda$.

of singular values of the matrix $L(f)$ that are larger than $|\lambda|$ or, equivalently, the number of negative eigenvalues of the following finite rank matrix,

$$R := |\lambda|^2 I - L(f)L^*(f) \ . \tag{18}$$

Rational allpass functions of a given degree k are members in the family H_k^∞ of all functions with k poles (or less) inside the unit circle, and whose magnitude is bounded on the unit circle, i.e., $\sup_{|z|=1} |f(z)| < \infty$. It turns out that the Schur-Cohn algorithm does not map the family H_k^∞ into itself, except when $k=0$. This means that this algorithm does not admit the same layer-peeling interpretation as the standard Schur algorithm. Nevertheless, we have found that it is possible to modify the Schur algorithm in such a way that the resulting recursion indeed maps the family H_k^∞ into itself and, therefore, admits the same layer-peeling interpretation as the classical Schur algorithm. Moreover, the layers involve only elementary (2×2) orthogonal and hyperbolic rotations (Ackner, Lev-Ari and Kailath (1989)).

Furthermore, our modified recursion applies to every function $f(z) \in H_k^\infty$, and not just to allpass functions. Each layer in the resulting transmission-line model has an sign or 'polarity' (Fig. 2). While a *p-type layer* (i.e., one of positive polarity) maps H_k^∞ into itself, a *n-type layer* maps H_k^∞ into H_{k-1}^∞ namely, it reduces by one the number of poles within the unit disc. Therefore, the number of n-type layers in the transmission-line model that is generated by our modified algorithm equals the number of poles that the function $f(z)$ has within the unit disc. This is the key idea in the construction of efficient procedures for zero-location and model-order reduction, and in the solution of several interpolation and extension problems arising in various control applications (Ackner, Lev-Ari and Kailath (1989)).

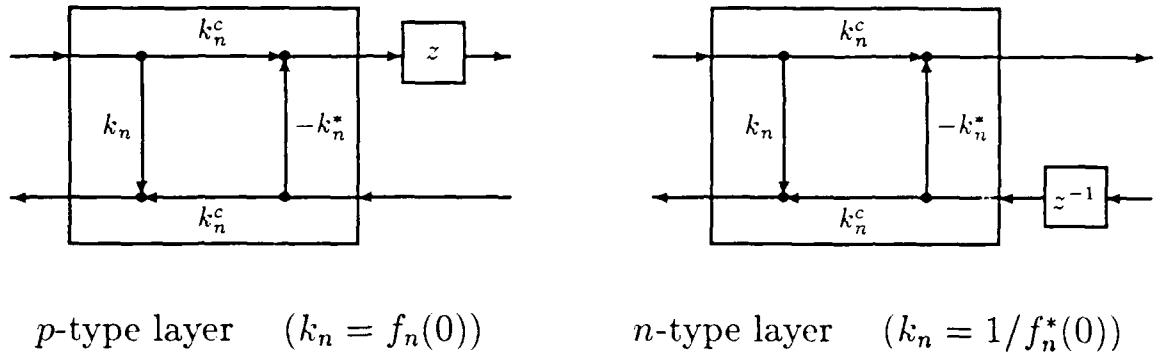p-type layer $(k_n = f_n(0))$ n-type layer $(k_n = 1/f_n^*(0))$

Figure 2. Layers for the modified Schur algorithm

Recursive layer peeling for a medium with multiple inputs and outputs involves a generalization of the Schur algorithm to *matrix-valued analytic* functions. One version of the matrix Schur algorithm (Delsarte, Genin and Kamp (1979)) requires hyperbolic *matrix rotations*, which are computationally expensive since they involve the finding of the square root of (positive definite) matrices. Moreover, this approach cannot be generalized to matrix functions with elements in H_k^∞, because now it would involve square roots of indefinite matrices.

An alternative approach to the *matrix analytic* case was taken by Dewilde and Dym (1981). They extract simpler layers than in the method of Delsarte, Genin and Kamp and, as a result, there is no need to compute square roots of matrices. This computational procedure, which is known as a *tangential Schur algorithm*, was the starting point for our research on layer-peeling methods for MIMO (multiple-input/multiple-output) systems.

We have shown that the procedure of Dewilde and Dym can be transformed into an equivalent form that involves only elementary (2×2) rotations. In fact, for $p \times q$

matrix scattering functions, the peeling of each layer in our version of the algorithm is implemented by a sequence of $q-1$ elementary orthogonal rotations, a single hyperbolic rotation, and a single block-delay (Fig. 3). These operations can be easily implemented in either software or hardware.
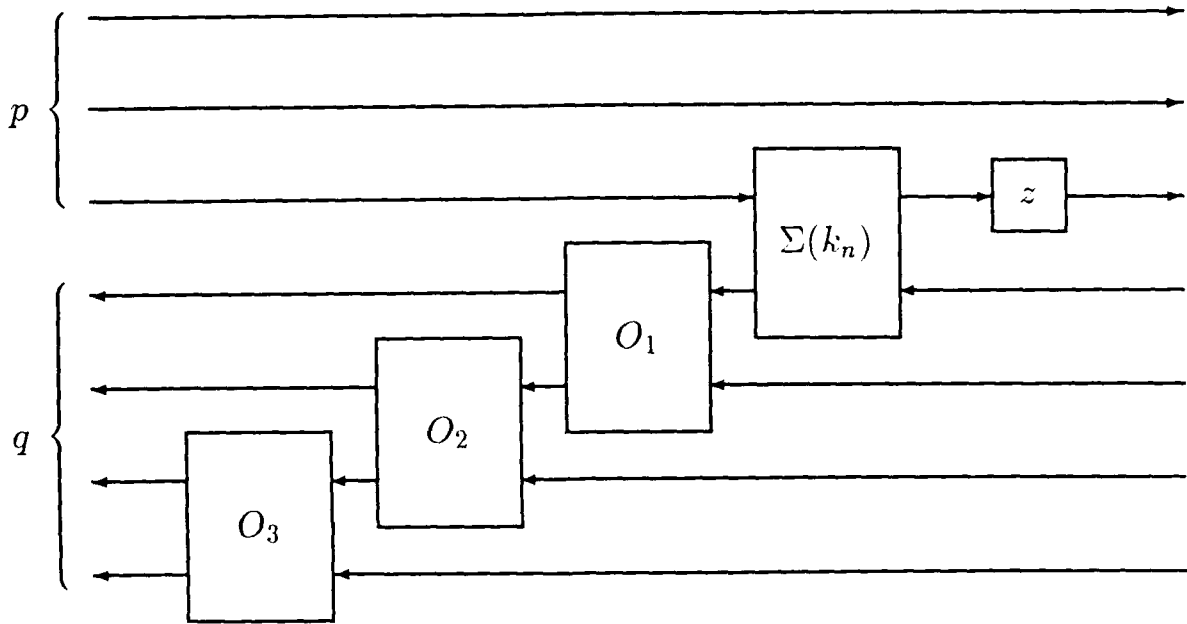

Figure 3. Single layer of the multichannel Schur algorithm $(p = 3, q = 4)$

In addition to requiring significantly fewer computations than the tangential Schur algorithm, our version also serves to clarify the relationship between the MIMO case and the better-known scalar or SISO (single-input/single-output) case. The MIMO procedure differs from the scalar one only by the presence of elementary orthogonal rotations. Thus both procedures share the same *core*, which consists of elementary hyperbolic rotations and delay elements. Consequently, we were able to show that our version of the tangential Schur algorithm can be modified to accommodate scattering

functions with poles within the unit circle, and that this modification affects only the 'core' part of the procedure (i.e., it is independent of the number of inputs or outputs). This means that each layer still has a 'sign' or polarity, as in the SISO case, and that the number of n-type layers coincides with the number of Smith-McMillan poles of the given matrix scattering function. This work is part of the dissertation research of R. Ackner, to be completed by June 1990 or earlier.

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I. Koltracht and T. Kailath, "Linear Complexity Parallel Algorithm for Discrete-Time Wiener Filters with Optimum Lag," *IEEE Trans. ASSP*, Vol. 36, no. 10, pp. 1680-1681, October 1988.

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Accepted Papers

A. Bruckstein, T. Kailath, I. Koltracht and P. Lancaster, "On The Reconstruction of Layered Media From Reflection Data," *SIAM J. Matrix Analysis*.

H. Lev-Ari, Y. Bistritz and T. Kailath, "Generalized Bezoutians and Families of Efficient Root-Location Procedures," *IEEE Trans. Circ. Syst.*

Published Conference Papers

T. Kailath and J. Chun, "Generalized Gohberg-Semencul Formulas," *International Conference on Operator Theory: Advances and Applications*, plenary lecture, Calgary,

Canada, August 22-26, 1988. Reprinted in *Advances and Applications*, dedicated to I. Gohberg, *Gohberg Anniversary Volume*, Vol. 40, pp. 231-246, Springer-Verlag, 1989.

Papers Under Review

H. Lev-Ari, K.-F. Chiang and T. Kailath, "Constrained-Input/Constrained Output Stability of Adaptive RLS Lattice Filters," *IEEE Trans. Circuits and Systems*.

A. Dembo and O. Zeitouni, "Maximum A-Posteriori Estimation of Random Fields - Part II: Non-Gaussian Fields," *Journal Multivariate Analysis*.

Y. Bistritz, H. Lev-Ari and T. Kailath, "Immittance-Domain Three-Term Schur and Levinson Recursions for Quasi-Toeplitz Complex Hermitian Matrices," *SIAM J. on Matrix Analysis and Appls.*.

J. Chun and T. Kailath, "Systolic Array Implementation of the Square Root Chandrasekhar Filter," *IEEE Trans. on Automa. Control*.

Presentations at Meetings and Symposia

Special Invited Lectures:

- Keynote Lecture, Sixth Army Conference on Applied Mathematics and Computing, Boulder, CO, May 31 - June 3, 1988
- Plenary Lectures, Summer Program on Signal Processing, Institute of Math and Its Applications, Minneapolis, MN, June 1988.
- Centennial Lecture, American Math Society, Society of Industrial and Applied Math, Minneapolis, MN, July 14, 1988
- Keynote Lecture, NATO Advanced Study Institute on Linear Algebra, Digital Signal Processing and Parallel Algorithms, Leuven, Belgium, August 1-12, 1988
- Plenary Lecture, International Conference on Operator Theory: Advances and Applications, University of Calgary, Alberta, Canada, August 21-26, 1988.
- Symposium on Applied Mathematics and Scientific Computing, Computer Science Department, Stanford University, April 21, 1989
- Special Lecture Series, Imperial College of Science Technology and Medicine, Department of Electrical Engineering, London, England, June 1-17, 1989, July 7 - August 11, 1989.
- Plenary Lecture, International Symposium on the Mathematical Theory of Networks and Systems (MTNS-89), Amsterdam, The Netherlands, June 1989

Conferences Attended:

- Indo-US Workshop on Systems and Signal Processing, Bangalore, India, January 1988
- 1988 IEEE Communication Theory Workshop, Sedona, AZ, April 1988
- International Conference on Acoustics, Speech and Signal Processing, New York, April 1988
- SIAM Conference on Applied Linear Algebra, Madison, WI, May 1988
- Systolic Arrays Conference, San Diego, CA, May 1988
- Sixth Army Conference on Applied Mathematics and Computing, Boulder, CO, May 1988
- SDIO Innovative Science and Technology Annual Information Processing Symposia, Arlington, VA, June 1988.
- SIAM Annual Meeting, Minneapolis, MN, July 1988
- Institute of Math and Its Application, Summer Program on Signal Processing, Minneapolis, MN, June-July 1988.
- NATO Advanced Study Institute, Leuven, Belgium, August 1-7, 1988.
- International Conference on Operator Theory: Advances and Applications, University of Calgary, Alberta, Canada, August 1988.
- International Conference on Linear Algebra and Applications, Valencia, Spain, September 1988
- 21st Asilomar Conference on Signals, Systems and Computers, Pacific Grove, CA, November 1988
- The 27th IEEE Conference on Decision and Control, Austin, TX, December 1988

Seminars

- June 1988, Penn State University, State College, PA
- July 1988, Fort Hunter-Liggett, King City, CA
- August 1988, Indian Institute of Science, Bangalore, India
- January 3, 1989, Institute of Telecommunication and Electronics Engineering, Bangalore, India
- January 4, 1989, Naval Physical and Oceanographic Laboratory, Cochin, India
- January 8, 1989 Indian Institute of Technology, Bombay, India
- February 3, 1989, Laboratorio de Computacao Cientifica, Rio de Janeiro, Brazil
- February 7, Universidad Nacional de La Plata, Argentina
- April 13, 1989, EE 201C Seminar, Stanford University
- April 28, 1989, High Noon Lecture Series, School of Engineering, Stanford University

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